plan of the magnetic rock causing the disturbance has been made, and, together with the complete set of observations, the following results have been deduced.

The cause of the disturbance is a ridge of rock permanently magnetized, repelling the north-seeking end of the needle. In the transverse section this ridge is steeper on the south-east side where the disturbances are at a maximum than on the north-west side. Longitudinally the ridge rises rather abruptly to a principal peak (as determined by the point of maximum disturbance of the vertical force), followed by a depression, and a second peak, finally falling abruptly to the level.

The principal values of the disturbances caused by this ridge are—

Declination......	56° E. on S.E. side, 26° W. on N.W. side.
Inclination	−29°.
Horizontal force..	−1·92 on S.E. side; +1·04 on N.W. side.
Vertical force ...	−4·44 metric units.

A geological survey of the coast at the Red Cliff (see map), where the greatest disturbances of the magnetic elements were observed, was made, and specimens of rock and sand were obtained which have since been tested for susceptibility. The evidence from these does not give any direct information tending to show the exact nature of the rock causing the remarkable disturbances over the magnetic shoal, but the character of the disturbances caused by the visible and invisible land are of a similar character.

Five diagrams are appended showing the data from which the foregoing results have been deduced, with a map showing the position of the "magnetic shoal" relative to the neighbouring land. Also a geological map of the Red Cliff and neighbourhood.

II. "A Dynamical Theory of the Electric and Luminiferous Medium. Part II; Theory of Electrons." By Joseph Larmor, F.R.S., Fellow of St. John's College, Cambridge. Received May 16, 1895.

(Abstract.)

In a previous paper on this subject,* it has been shown that by means of a rotationally elastic æther, which otherwise behaves as a perfect fluid, a concrete realization of MacCullagh's optical theory can be obtained, and that the same medium affords a complete representation of electromotive phenomena in the theory of electricity. The ponderomotive electric forcives were, on the other hand, deduced

* 'Roy. Soc. Proc.,' November, 1893; 'Phil. Trans.,' 1894, A, pp. 719—822.

from the principle of energy, as the work of the surplus energy in the field, the motions of the bodies in the field being thus supposed slow compared with radiation. It was seen that in order to obtain the correct sign for the electrodynamic forcives between current systems, we are precluded from taking a current to be simply a vortex ring in the fluid æther; but that this difficulty is removed by taking a current to be produced by the convection of electrons or elementary electric charges through the free æther, thus making the current effectively a vortex of a type whose strength can be altered by induction from neighbouring currents. An electron occurs naturally in the theory as a centre or nucleus of rotational strain, which can have a permanent existence in the rotationally elastic æther, in the same sense as a vortex ring can have a permanent existence in the ordinary perfect fluid of theoretical hydrodynamics.

In the present paper a further development of the theory of electrons is made. As a preliminary, the consequences, as regards ponderomotive forces, of treating an element of current $\iota\delta s$ as a separate dynamical entity, which were indicated in the previous paper, are here more fully considered. It is maintained that a hypothesis of this kind would lead to an internal stress in a conductor carrying a current, in addition to the forcive of Ampère which acts on each element of the conductor at right angles to its length. Though this stress is self-equilibrating as regards the conductor as a whole, yet when the conductor is a liquid, such as mercury, it will involve a change of fluid pressure which ought to be of the same order of magnitude as the amperean forcive, and therefore capable of detection whenever the latter is easily observed. Experiments made by Professors FitzGerald and Lodge on this subject have yielded purely negative results, so that there is ground for the conclusion that the ordinary current-element $\iota\delta s$ cannot be legitimately employed in framing a dynamical theory.

This result is entirely confirmed when we work out the properties of the field of currents, considered as produced by the convection of electrons. There can be no doubt that a single electron may be correctly taken as an independent element of the medium for dynamical purposes; so that electrodynamical relations deduced from a statistical theory of moving electrons will rest on a much surer basis than those derived from the use of a hypothetical current-element of the ordinary kind, in cases where they are in discrepancy.

Now it is shown that an intrinsic singularity in the æther, of the form of an electron e, moving with velocity $(\dot{x}, \dot{y}, \dot{z})$ relative to the quiescent mass of æther, is subject to a force e (P, Q, R), given by equations of the form

$$\mathrm{P} = c\dot{y} - b\dot{z} - d\mathrm{F}/dt - d\Psi/dx;$$

in which (a, b, c) is the velocity of flow of the æther where the electron is situated, and is equal to the curl of (F, G, H) in such way that the latter is Maxwell's vector potential given by the formulæ of the type

$$F = \int \frac{u}{r} d\tau + \int \left(B \frac{d}{dz} - C \frac{d}{dy} \right) \frac{1}{r} d\tau ;$$

and where Ψ is the electrostatic potential due to the electrons in the field, so that $\Psi = c^2 \Sigma e/r$, where c is the velocity of radiation. These equations are proved to hold good, not merely if the motions of the electrons are slow compared with radiation, as in the previous paper, but quite irrespective of how nearly they approach that limiting value; thus the phenomena of radiation itself are included in the analysis.

An element of volume of an unelectrified material medium contains as many positive electrons as negative. This force (P, Q, R) tends to produce electric separation in the element by moving them in opposite directions, leading to an electric current in the case of a conductor whose electrons are in part free, and to electric polarization in the case of a dielectric whose electrons are paired into polar molecules. In the former case, the rate at which this force works on a current of electrons (u', v', w'), is $Pu' + Qv' + Rw'$; it therefore is identical with the electric force as ordinarily defined in the elementary theory of steady currents. In the case of a dielectric it represents the ordinary electric force producing polarization. So long as a current is prevented from flowing, the ponderomotive force acting on the element of volume of the medium is the one of electrostatic origin due to such polarization as the element may possess, for as the element is unelectrified it contains as many positive electrons as negative. But if a current is flowing, the first two terms of (P, Q, R), instead of cancelling for the positive and negative electrons, become additive, as change of sign of the electron is accompanied by change of sign of its velocity; so that there is an electrodynamic force on the element of volume,

$$(X, Y, Z) = (v'c - w'b,\ w'a - u'c,\ u'b - v'a),$$

where, however, (u', v', w') is the *true* current composed of moving electrons, not the total circuital current (u, v, w) of Maxwell, which includes the rotational displacement of the free æther in addition to the drift of the electrons.

The electric force (P, Q, R) as thus deduced agrees with the form obtained originally by Maxwell* from the direct consideration of his concrete model of the electric field, with idle wheels to

* Maxwell, "On Physical Lines of Force," 'Phil. Mag.,' 1861–62; 'Collected Papers,' vol. 1, pp. 450—512.

represent electrification. It has been pointed out by von Helmholtz and others, that the abstract dynamical analysis given in his *Treatise* does not really lead to these equations when all the terms are retained; this later analysis proceeds, in fact, by the use of current-elements, which form an imperfect representation, in that they give no account of the genesis of the current by electric separation in the element of volume of the conductor.

The ponderomotive force (X, Y, Z) is at right angles to the direction of the true current, and is precisely that of Ampère in the ordinary cases where the difference between the true current and the total current is inappreciable. It differs from Maxwell's result in involving true current instead of total current; that is, the forcive tends to move an element of a material body, but there is no such forcive tending to move an element of the free æther itself. In this respect it differs also from the hypothesis underlying von Helmholtz's recent treatment of the relations of moving matter to æther.

When we treat of a single electron, (a, b, c) is the flow of the æther where it is situated. When we treat of an element of volume with its contained electrons, (a, b, c) becomes the smoothed out, or averaged, flow of the æther in the element of volume; it is circuital because the æther is incompressible, and thus it represents the magnetic induction of Maxwell.

When magnetic polarization of the medium contributes to the forcive, it is necessary to divide (a, b, c) into two parts, one part (α, β, γ) contributed by the medium as a whole, and independent of the surroundings of the element, and the other representing the effect of the polarization in the immediate neighbourhood; the former part is, of course, the magnetic force of Maxwell. Similar considerations apply as regards the electric force in a polarized dielectric; it is clearly proper to define it so as to correspond to magnetic induction, not to magnetic force. It is then shown from the direct consideration of the orbital motions of electrons, that there is, in addition to the electrodynamic force on the element of volume of the material medium, a magnetic force derived from a potential function $\frac{1}{2}\kappa(\alpha^2+\beta^2+\gamma^2)$, and a force of electric origin derived from a potential $(K-1)/8\pi c^2 \cdot (P^2+Q^2+R^2)$. If the element carries an electric charge of density ρ, there is also the force $\rho(P, Q, R)$. In addition to these latter forces on the polarized element, there are also stresses due to interaction between neighbouring parts, in which are to be found the main explanation of the phenomena of electrostriction and magnetostriction.

As an example of these ponderomotive forces, the mechanical pressure produced by radiation is examined later on, with a result half that of Maxwell when the light is incident on an opaque body, and which gives pressures on the two sides of the interface each equal to Maxwell's

expression multiplied by $\frac{1}{2}(1-\mu^{-2})$, when the interface separates two transparent media.

The distinction between true current and total current is practically immaterial, except in questions relating to electrical vibrations and to optics. The remaining part of the theory is therefore developed more particularly with a view to optical applications. At the end of the previous paper a brief outline of the method of treating optical dispersion was given, and it was shown that the same principles led directly to Fresnel's formula for the effect on the velocity of light produced by motion, through the æther, of the material medium which transmits it. In the latter respect the theory is in agreement with a more recent discussion by H. A. Lorentz, of the propagation of electrical and optical effects through moving media.

A detailed theory of optical propagation in transparent and opaque ponderable media is given, on the basis that it is the contained electrons that are efficient in modifying the mode of propagation from that which obtains in free æther. The dispersive theory of MacCullagh had been physically interpreted in the earlier part of the previous paper; but it appears from the same train of reasoning as was there applied to Cauchy's theory, that molecular magnitudes are too small compared with the wave-length to allow any considerable part of the actual dispersion to be accounted for statically in that way. The rotatory dispersions, both natural and magnetic, are, however, structural phenomena; and this accounts for their smallness compared with ordinary dispersion.

As regards ordinary dispersion, a formula is obtained for the case of perfectly non-conducting media, namely, $\mu^2 = 1 + A/(\beta^2 - p^2)$, where $2\pi/p$ is the period, of the same type as one recently deduced by von Helmholtz by an abstract process based on the principle of Least Action combined with a theory of electrons, which, however, does not correspond with the views here developed. That this formula is a good representation of the experimental facts for ordinary transparent media is generally recognised; especially as it may, in case of necessity, be modified by the inclusion of slight non-selective opacity, due to drift of free electrons, after the manner of ordinary conduction. When this kind of general opacity is predominant, the result obtained in the paper conforms to the main features of metallic propagation; thus, with sufficient conductivity the real part of the square of the refractive index becomes negative, and the real part of the index itself may become less than unity, while the dispersion is usually abnormal.

When the phenomena of moving media are treated, dispersion may, for simplicity, be left out of account. It is shown that, if the view described in the previous paper, that all the dynamical properties of matter are to be derived from the relations of electrons, with or

without intrinsic inertia, in a rotationally elastic fluid æther, is entertained, the null result of the Michelson-Morley second-order experiment on the effect of the Earth's motion on the velocity of light becomes included in the theory; in fact, according to a suggestion thrown out by FitzGerald and Lorentz, and developed somewhat in this manner by the latter, the second-order optical effect is just compensated by a second-order effect on the lengths of the moving arms of Michelson's apparatus, which is produced by its motion along with the Earth through the æther.

As mixed dynamical and statistical theories of electrons or other objects require delicate treatment, especially when pushed, as here, to the second order of small quantities, the formulæ of this part of the paper are deduced independently by two very different analytical methods. In the first place, there is the usual process of extending the fundamental circuital relations of the free æther which express its dynamical relations as differential equations of the first order, by suitable modification of the significance of the vectors involved in them, so that the same equations shall apply to ponderable media as well, the vectors then representing averages taken over the element of volume. The other method consists in working out the dynamics of a single electron, and applying the results statistically to the inclusion of the various ways in which the electric current arises from the movements of the electrons in ponderable media.

The theory as thus developed from the electron as the fundamental element, may be stated in a form which is independent of the dynamical hypothesis of a rotational æther. Maxwell's formal equations of the electric field may take the place of that hypothesis, though it may, I think, be contended that an abstract procedure of that kind will neither be so simple nor so graphic, nor lend itself so easily to the intuitive grasp of relations, as a more concrete one of the type here employed.

The exact permanence of the wave-lengths in spectra, under various physical conditions, may be ascribed to the influence of radiation on the molecule, which keeps it in, or very close to, a constant condition of steady motion, of minimum total energy corresponding to its predetermined constant momenta. It is also pointed out, from the analogy of physical astronomy, that the harmonic oscillations into which the spectroscope divides the radiation from a molecule, may be far more numerous than the co-ordinates which specify its relative motions; that, therefore, relations of a semi-dynamical character may be discovered among the spectral lines, without its being rendered likely that we can ever penetrate from them back to the actual configuration of the molecular system.

Reverting finally to matters relating purely to a rotational æther theory, with electrons as the sole foundation for matter, it is possible to

identify the inertia of matter with the electric inertia of the electrons, if only we may assume their nuclei to be small enough, or sufficiently numerous. And the fact that these nuclei have free periods of elastic radial vibration in the fluid æther, not subject to damping by radiation, reminds us that a pulsatory theory of gravitation has been developed by Hicks and Bjerknes. There is no recognised fundamental interaction of electric and radiative phenomena with gravitation, so for present purposes we are not bound to produce a precise explanation of gravitation at all. The scope of this remark is restricted to merely showing that a rotational æther is not incompetent to include such an action among its properties.

III. "On the Refractive Index of Water at Temperatures between 0° and 10°." By Sir JOHN CONROY, Bart., F.R.S., Fellow and Bedford Lecturer of Balliol College, and Millard Lecturer of Trinity College, Oxford. Received May 16, 1895.

In 1856 Jamin ('Comptes Rendus,' vol. xliii, p. 1191) published an account of observations he had made on the refractive index of water at temperatures between 30° and 0°. He used an interference method, and found that as the water cooled the index increased.

"La masse totale de l'eau qui d'abord était à 12 degrés, se refroidissant continuellement, arriva bientôt à 4 degrés, c'est-à-dire, au point où les variations de l'indice devraient changer de signe, et où le déplacement des franges devrait être inverse. Mais rien de pareil ne se montra, et en continuant le refroidissement jusqu'à zéro, on continua d'observer une augmentation de l'indice. Il n'y a donc pas de maximum dans la valeur du coefficient de réfraction quand il y en a un dans la densité."

In another experiment the temperature of the column of water through which one of the beams of light passed was kept at 0°, whilst that of the other was gradually raised to 30°; he found by the displacement of the bands that the index decreased steadily. He did not, apparently, publish any numerical values for the indices, but states that they are accurately given by the empirical formula $K_t = K_0 - (0{\cdot}000012573)t - (0{\cdot}000001929)t^2$.

Two years later Gladstone and Dale ('Phil. Trans.,' 1858, p. 887) gave an account of observations that they had made "on the influence of temperature on the refraction of light;" they used a hollow glass prism, and determined the angles of minimum deviation for water, and several other liquids, at various temperatures. They say, "our determinations were performed repeatedly and most carefully on water near the freezing point; they confirm the observations